AYDA

I hope to produce a series of Douglas Leedy's miscellaneous essays, mostly on musical topics, over the next few months, to join those already published on line: Singing Ancient Greek [https://escholarship.org/uc/item/1rj4j3n1], Recognizing the Midtone, a Primary Musical Interval [www/shere.org/Leedy/Midtone.pdf].

The significance and excellence of his work speaks for itself, I hope, even in publications with which he would perhaps have some reservations were he still with us. The clarity and elegance of his handwriting, from which many of these essays will be transcribed, cannot be reproduced in print, as will be seen in the musical examples, reproduced here in facsimile from his work.

*The present work is from a paper clearly labeled "*draft*" and bearing some illegible, clearly subsequently pencilled annotations. I am sorry not to have managed to issue these during Leedy's lifetime: he would surely have had suggestions for improvement. I post these essays on line free of copyright, as he would have wanted, and welcome comment.*

— Charles Shere
info@shere.org
March 28 2018,
the third anniversary of Leedy's death

proof edition

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MUSIC TO THE POWER OF 4

Douglas Leedy (1938-2015)

FOUR is "the number of material order"*: four elements, four compass points, four "temperaments," four seasons and quarters of the day, four moon phases, four dimensions of space-time, and so on. Our physical space is more often than not defined by four walls; surveyors lay out four-sided tracts of land. Similar are the playing areas for most sports, the baseball diamond most obviously, perhaps, as well as game boards—chess, for example. (Not to forget. That there are four suits in thee standard card-deck, the cards of which — triskaidekaphobes must know this! — number 4 x 13.

This ordering power of four extends strongly into music: four voices in the standard harmonic arrangement ("SATB") and string quartet; four movements to a symphony; four sections I the 19th-century stereotype of the sonata-allegro form (exposition-development-recapitulation-coda). *Vierhebigkeit* ("for-beat-ness") was the notorious imposition on some medieval musical repertoires by a few German musicologists of a century ago (Hugo Riemann in particular) of 4-beat, 4-bar rhythmic organization — since discredited.

Vierhebigkeit is nevertheless fundamental to the standard European-American music of recent times, popular and classical : it is the number of bars in a stereotypical musical phrase, as well as the number of beats in a bar of "common time" (**C** or 4/4), which consists

* *The Mystery of Numbers* by Annamarie Schikurmel (Oxford, 1993), p. 86; this is a useful and interesting, though at times careless, inaccurate (and incomplete) compendium.

of four "quarter notes," the basic temporal unit in Western musical notation since the 18th century.*

In pitch structures, the musical interval of the fourth manifests itself in two ways: first, it is defined as an interval spanning four consecutive degrees in a diatonic scale (and is subject to some variation in absolute size for reasons given below).† This melodic interval of a fourth — 1-2-3-4, *re-mi-fa-sol*, *d-e-f-g*, *sa-ri-ga-ma* are all examples of a fourth in different systems of scale-degree names — is a strong presence in music around the planet.

Second, the perfect fourth is found acoustically in the harmonic series as the third simplest interval, following the octave and perfect fifth: it is located between the third and fourth harmonics. If one divides a monochord string into four equal segments, the pitch of a length of the string equal to three of these segments will be precisely a fourth above that of the whole string, showing the ratio of the interval, 4 to 3.

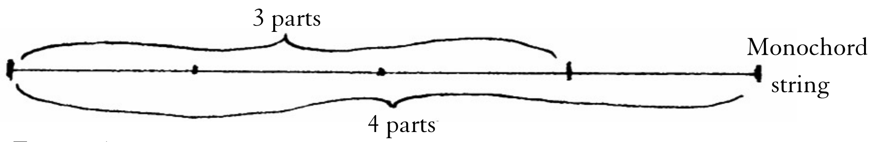


FIGURE 1

In the diatonic scale, the perfect fourth consists of two tones and a diatonic semitone. Because of the intervallic structure of that scale, however, one of the seven fourths that one can construct on each of its seven degrees is quire different from the others: he scale's three consecutive tones produce an augmented fourth, the infamous "tritone," lager than the perfect fourth by a chromatic semitone, an interval that—unlike the perfect fourth—can vary considerably in its intonational shading. We will have to return later to this mysterious

* The prime numbers 2, 3, 5 and 7 are also of great significance musically.

† In the semitone-unit system of interval definition (associated with the Princeton theorists), the perfect fourth is five units in width, its semitone content in 12-tone equal temperaments. The term "fourth," however, is valid not only in that system but in the different Western historical tunings as well.

interval (which can be found in the example of two-voice organum, below, p. 11). [*ed. note: page references irrelevant in ebook*]

Chromatic alteration of a diatonic scale degree can result in altered fourths of various kinds, the two most commonly found of which are the augmented fourth, above, and the diminished fourth, an interval a chromatic semitone narrower than a perfect fourth. (Some examples of this relatively rare interval are given below.) The diminished fourth exists in at least two intonational shades, the very dissonant 25:16 (an octave less two pure major thirds in just intonation and quarter-comma meantone temperament), and the purely nominal one-third octave interval of 12-equal, identical to the major third. The *floruit* of this interval seems to be the 17th century, where its expressive qualities were particularly appreciated and displayed—in the true, dissonant tuning, needless to say.

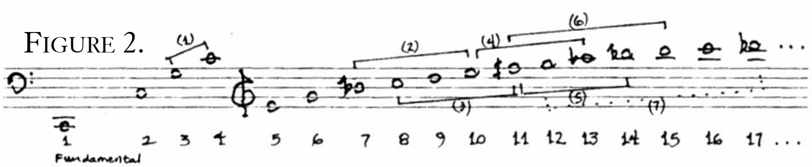
C. Monteverdi: Madrigali, Libro quinto. 1605 "Ma tu, più Chee mai dura," 12 semibreves before end (X diminished fourth). Note the movement of the quinto and alto voices of a diatonic and chromatic semitone respectively: in just intonation the latter is nearly a quarter tone narrower than the former.

G. Frescobaldi: Il Secondo Libro di toccate etc. (1637) Canzona Quarta, bar 11

H. Purcell: Dido and Æneas ((1689)
Final aria (“When I am laid in earth”), bar 7, 17 etc.

The last of the four fundamental forms of he interval of a fourth is the semiaugmented (or “plus”) fourth, an interval rare in Western music, but not so in much of the music of the non-Western world. It consists of two tones and a midtone (¾ tone), and it is most simply derived from the relationship of the 11th harmonic to the triple octave, 8, of the fundamental, or 11:8 (Figure 2, below). While not diatonic intervals strictly speaking, they participate in what I refer to as midtone diatonic scales, and are basic to the melodic modes (*magãmãt*, *dastgãhãb*, *makamlar*) of classical and popular music of the Middle East and North Africa.*

FIGURE 2.



Harmonic Series of Tones on Fundamental C, to Harmonic 17,
Showing Primary Intervals of a Fourth

flat by an additional 1/8 tone, approximately
about a quarter tone sharp and flat, respectively

Any two pitches in a harmonic series whose number -ratio can be reduced to 4:3 will form a perfect fourth: 12 and 9, or 20 and 19, for example.
(1) Perfect 4th, 4:3
(2) Septimal augmented 4th or tritone, 10:7
(3) Plus or semiaugmented fourth, 11:8

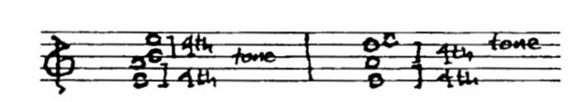
* See Leedy: “Recognizing the Midtone, a Fundamental Musical Interval” www.shere.org/Leedy/Midtone.pdf

(4) Minus or semidiminished fourth, 13:10
(5) Diminished fourth, 14:11
(6) Plus or semiaugmented fourth (narrower than (3)), 15:11
(7) Diminished fifth, 17:12, which inverts to the augmented 4th 24:17; these intervals are close to equal in size and divide the octave almost exactly in half.

In its function as subdominant harmony, the fourth degree of the scale is as stable as the fifth. When the fourth degree is subordinated to the fifth in a dominant seventh chord, however, its intonation is unstable. As a minor seventh above the fifth degree the fourth has three theoretical values: a 16:9s seventh, giving the normal 4:3 fourth; a higher seventh of 9:5 (where the fourth is a pure 6:5 minor third above the second degree), giving a fourth of 27:20; and a much lower pitch as the harmonic 7th (7:4) of the dominant, with a 21:16 fourth.

Note that the presence of 4 in the designation of the interval of a fourth and in the 4:3 ratio is coincidental: both the fourth and fifth, as one can see above, result from the appearance of the number 3, which makes a new pitch between harmonics 2 and 4; powers of 22 are all octaves of the fundamental.

The ancient Greek theorists of music called the fourth *diatessaron*, meaning literally "through four [pitches or lyre-strings]." The fourth was the framework of the tetrachord ("four [lyre-]strings"0, and was the organizing interval — more importantly than the fifth or octave — of ancient Greek music in general. The octave they divided into two fourths and a tone, the defining pitches of which were considered fixed. The tetrachords could be conjunct — with a pitch in common — or disjunct, as shown below; within the tetrachord, movable pitches determined the nature of the genus (enharmonic, chromatic, diatonic), something not entirely unlike what happens in the tetrachords of our diatonic modes, the interval-orderings of which are, as it happens, four in number: tone-tone-semitone; tone-semitone-tone, semitone-tone-tone, and, of course, tone-tone-tone, the inescapable augmented fourth, a prominent feature of the Lydian mode (the natural notes from F to F).



Fourths: disjunct and conjunct, in the octave of ancient Greek theory

> The Greeks' division of the octave into two fourths separated by a tone can be expressed in the proportions 6:8:9:12*, which provides a demonstration of two of the three most important of their mean proportionals, the arithmetic and the harmonic.† (Third is the geometric mean, which we will see later.) The arithmetic mean is the "average" of two numbers: in the present case he numbers are 6 and 12; we add them and divide by 2, (6+12/2 = 9. On a monochord this gives the interval of a fourth above the pitch of the longer string, i.e., 12:9 or 4:3. The harmonic mean is a similar proportional of the reciprocals of given umbers: here our reciprocals will be 1/6 and 1/12. By the formula for finding the harmonic mean we arrive at the number 2(6x12)/6+12 = 144/18 = 8. In terms of the monochord string-length of 12, we find we have the interval 12:8 or 3:2, a fifth, as the harmonic mean.‡

The "Greek" division of the octave occurs frequently in melody (for example, in the excerpt below from Brahms). The basic framework of blues melody (as I hear it) is identical: two (descending) fourths separated by a whole tone, where each fourth is basically a trichord of a midtone and mid third, from the top down, intervals with a considerable range of expressive intonational latitude.

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*These four points define the ancient Greeks' musical space, and their four principal intervals, the octave (*diaasōn*), fifth (*diapente*), fourth (*diatessoron*) and whole tone (*epogdoon*).

† For the Geeks' different mean-0proportional calculations — they recognized ten — see *A History of Greek Mathematics* by Thomas Heath (1921; reprint: Dover, 1981), vol. 1, p. 85-6 {a book both theoretical and practical}.

‡ See addendum, p. 20XXX

In melody the fourth can be seen at nearly every turn in the music of many cultures as a shaping interval — in Western melody perhaps as influential as the triad in this function. ("Shaping" interval is quite different from "structural" interval.) In the following musical examples one can see this effect at work in excerpts chosen rather unsystematically. These examples come mostly from re3cent times, but a glance at earlier monophonic or polyphonic music will immediately show the melodic importance of the fourth, particularly I fixing the points where melody changes direction. (We should keep in mind that in stepwise diatonic melody it is three melodic moves in one direction that make up the interval of a fourth: there seems to be an inherent desire in melody to turn after three moves in one direction, and the "turning points" of the fourth itself create a strong impression of the interval.)

J.S. Bach: B-minor Mass, "Dona nobis pacem" (1735)

W.A. Mozart: Symphony no. 41 in C major, K.551, finale (1788)

F.J. Haydn: The Creation, no. 13, "The heavens are telling" (1796-8), beginning, chorus soprano; bar 22, Gabriel

L. Van Beethoven: "King Stephen" Overture, op. 117 (1811), beginning; more than two fourths, as here, disrupt a sense of tonality (cf. Schoenberg, p. 10)

L. Van Beethoven: Sonata, op. 109 (1820), mvts. 1 and 3

H. Berlioz: Symphonie fantastique, op. 14 (1830), first mvt., bars 71ff

J. Brahms: "Es tönt ein voller Harfenklang," op. 17, no. 11 (1859-60), for women's voices, horn and harp, beginning

E. Satie: Gymnopédies 1 and 2 (1888), each from bar 5

E. Elgar: “Enigma” Variations, op 36 (1898-90, beginning of Var. 7

J. Sibelius: Symphony NO. 4, op. 63 (1911), beginning of 2nd mvt.

A. Schoenberg: Kammersymphonie, op. 9 (1906), from bar 5*

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* On the importance of the fourth in this work, see W. Frisch, *The Early Works of Arnold Schoenberg 1893-1908* (Univ. Of California Press, 1993), ch. 9, p. 220-47.

A. Berg: Symphonische Stücke aus der Oper "Lulu" (1935): Adagio, bars 75-6, rehearsal number 905, horns in F

B. Britten: Hymn to St. Cecilia (Auden), op. 27 (1942), SSATB, opening

P. Hindemith: "Aspparebit repentina dies" (1947), for mixed chorus and brass ensemble: First mvt., from bar 5; third mvt., from bar 44

Spike Jones and His City Slickers: "Chloe" (1946), transcr. from recording

As a harmonic interval, the fourth has been subjected to a millennium of confusion and discomfort in the West among composers and music theorists. Western music theory has had to maintain for centuries the artistic fiction that the perfect fourth is a harmonic dissonance, whereas it is in absolute acoustic terms the third-best consonance, as we already have seen, after the octave and fifth. There is a considerable history to this contradiction: Western harmony began by accepting — briefly — the fourth as consonant in the earliest two-part organum:

Example of simple organum at the fourth. *Musica Enchiriadis* (ca. 900 CE) (+ indicates tritone intervals).*

But as the major triad became an increasingly central sonority the fourth came to be regarded as unstable (as one sees in later theories of intervallic and triadic "roots") and was relegated to isolation in upper voice-parts. (It is one of the ironies of this conceit that the second inversion, or "6-4" disposition of the close-position major triad is the most consonant acoustically.)

Renaissance theorists differed in attitude toward the fourth. Zarlino wrote that "to some. It may appear novel that I include the fourth among the consonances,"† while Thomas Morley argued, reflecting the common opinion of his time, "But why they [i.e., the Greeks and Boethius] should make *diotessaron* a Consonant, seeing it

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* G. Reese: *Music in the Middle Ages* (Norton, 1940), Ex. 67, p.254.

† G. Zarlino: *Le Istitutioni harmoniche*, 1556, in *The Art of Counterpoint*, transl. by G.A. Marco and C. Palissca (1968); reprint, Norton, 1976), p 12

mightily offendeth the eare, I see no reason…"* J.J. Fux called this "a famous and difficult question"; "[Padre] Martini, basing his opinion upon that of Zarlino, goes so far as to call the fourth a perfect consonance."†

Ibn Sīnā, the Arab physician and music theorist of the 11th century, wrote (under the influence of the ancient Greeks) that "The noblest consonances are the large intervals, and among these, the octave and the fourth are the best."‡Leonhard Euler's theory of consonance makes the fourth next best after the fifth,§ and for Helmholtz the fourth is an "unqualified consonance."**

It was not until late in the 19th century that the perfect fourth came once again to be respected as an autonomous interval, and even as a consonance. As it appears to me, the composer who rediscovered the fourth's unique and excellent harmonic qualities was Erik Satie (1866-1928), who for whatever reason — the interval clearly appealed to his ear — made the fourth a characteristic feature of his musical style. Satie seems also to have been the first to use the fourth as a chordal building-block, analogous to the thirds in triadic harmony, and if so, it is he who opened the door to quartal harmony, which became a prominent feature of 20th century music, from Skryabin's "mystic chord" (below) to the chordal sonorities of Hindemith and jazz. The bold quartal chords (including the coloring effect of

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* T. Morley: *A Plaine and Easie Introduction to Practicall Musicke* (London, 1597; facsimile reprint, Gregg International, 1971), "Annotations Upon the second Part: to p. 70, vers. 29, lines 8-9 (no pagination).

† Alfred Mann, translator and editor, *Steps to Parnassus, by Johann Joseph Fux* (Norton, 1943), p.20-1, footnote 1.

‡ Quoted ihn. Ut Sachs: *The Rise of Music in the Ancient World*… (Norton, 1943), p. 289.

§ L. Euler: *Tentamen novae theoriae musicae* (St. Petersburg, 1739; facsimile reprint, Broude Bros., 1968), p. 61, 63.

** H. Helmholtz: *On the Sensationss of Tone*… 2nd English ed., teansl. By A.J. Ellis ([1885]; reprint, Doveer, 1954), p. 194, 196-7. Consonance of second inversion of major triad, p.219-20.

augmented fourths) of Satie's music for "Le fils des étoiles" (1891) are a truly remarkable development in Western music, and, so far as I am aware, unforeseen and unadumbrated.

E. Satie: Le fils des étoiles (1891, publ. 1896), from the 13th quarter-note

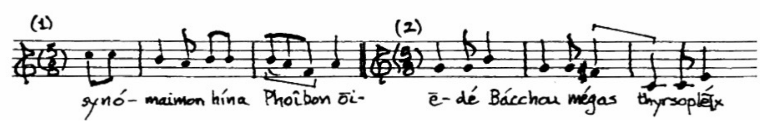
The growing use of fourths in chord-building resulted in part, I believe, from the souring of the thirds of triadic harmony by equal temperament; in addition, quartal "triads" are quite well suited to the hand on a keyboard instrument — especially the piano, with its long raised keys.

E. Satie: Socrate (1916-17), 3rd mvt., final bars

E. Satie: Troisième Nocturne (1919)

A. Skryabin: Prométhée, le poème du feu, op. 60 (1908-10), opening chord; the "mystic" chord, which evidently appears first in the fifth piano sonata, op. 53 (1907), bar 264

We return, finally, to that most elusive, yet somehow seductive, member of the family of fourths, the tritone, or augmented Fourth. The ancient Greeks seemed to enjoy the melodic tritone (the Plutarchian treatise on music quotes Aristoxenus's description of the "discovery" of this interval by the4 legendary musician Olympus, who "was struck by the beauty of the character of this procedure" — i.e., a particular melodic leap giving a tritone).* The interval is not uncommonly outlined in Gregorian chant melisma, as in the brief examples (in several different modes) here.



(1) Athenaeus: Paeasn ("Delphic Hymn"); (2) Limenius: Paean and Processional (127 BCE)†

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* Pseudo-Plutarch 11, 1134f, in *Gereek Musical Writings* I, edited and translated with commentary by A. Barker (Cambridge, 1984), p.216.

† The two Delphhic Hymns can be found transcribed in *Ancient Greek Music* by M.L. West (Oxford, 1992), p. 288, 296.

Gregorian Chant: (1) mode 8 (*g* finalis), *Liber Usualis* p. 600; (2) mode 3 (*f* finalis), p.1058; (3) mode 3 (*er* finalis), p. 1549

The harmonic tritone was tolerated very briefly in early polyphony, as we saw in the example of organum above (p. 11). But soon it became reviled in the Christian West, and remained so, a sound to be shunned or concealed, for centuries.

In the 20th century the unique sound of the augmented fourth became a positive virtue in different musical styles, not least because of its ambiguous quality; the tritone, too, is one half of a whole-tone scale such as one finds as something of an early 20-century musical fetish. The following examples re typical of the tritone's evocative power.*

C. Debussy: La mer (1903-5), 2nd mvt., bars 9-10

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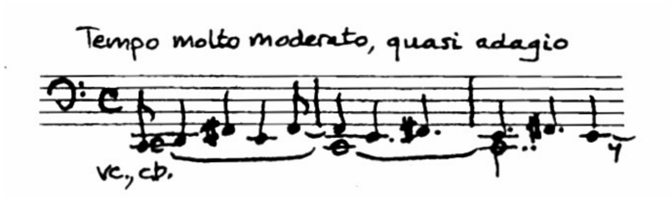
* Brief history of the tritone in the West, with examples and short bibliography, by W. Drabkin, "Tritone," *New Grove Dictionary of Music and Musicians*, 2nd ed. ed. by S. Sadie and Q. Tyrrell (Macmillan, 2001).

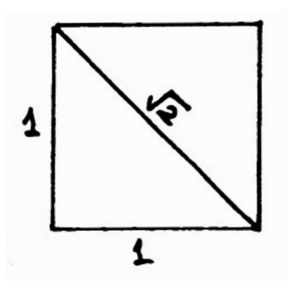


J. Sibelius: Symphony no. 4, op. 63 (1911), opening of first mvt.'(Sibelius built much of this work on the tritone; cf. the ex. above, p.9)

The 72 heptatonic scales of the *malakarta* system of classical South Indian music are divided into two basic groups: 36 with a "natural" Fourth degree (*ma*) of 8:3 (*komal* or "soft" *ma*) and 36 with an augment3ed fourth (*tivra* or "sharp" *ma*), the intonation of which can vary somewhat.

But the tritone is also a sonorous metaphor that carries us into mathematics, geometry and even ethics. In our equal-temperament system the tritone is exactly half an octave. We know that the ratio of an octave — string length or frequency — is 2:1, so if we are to find one half of that interval we need to know what quantity multiplied by itself will give us the number 2; that quantity turns out to be the square root of 2, $\sqrt{2}$, an irrational number, i.e., one that cannot be expressed as the ratio of two whole numbers. The ancient Greeks were familiar with irrational numbers (which they called "incommensurables" [*asymmetroi*]), the simplest of which is in fact $\sqrt{2}$, and they knew this quantity in its most basic form as the length of the diagonal of a square with a side of 1. l(This follows from the Pythagorean theorem for a right triangle: $\sqrt{12} + 12$.)

FIGURE 3



The various tunings of the tritone,* when expressed as numerical ratios, were — as the Greeks knew — more or less close approximations of the actual value of $\sqrt{2}$ (which we know as an endless decimal, 1.41421356…). The oldest tritone likely is the complex-looking Pythagorean ratio of 729:512 (made up of three tones all of the ratio of 9:8, or $(9:8)^3$). The tritone wit the simplest ratio is composed of the unequal tones 8:7, 9:8 and 10:9 (which decrease in size; see the harmonic series in Figure 2, above), and has the ratio 10:7; it is a surprisingly consonant-sounding, stable — and one might say pleasing — interval.

> The interval $\sqrt{2}$:1 gives us the third of the three mean proportionals important in music: it s the geometric mean† of 2:1. Expressed as decimal ratios, 729:512 is 1.4283, and 10:7 is 1.42857. The "just-intonation" tritone (where intervals are based on the primes 2, 3 and 54 only), 45:32, consists of two 9:8 tones and one of 10:9; its decimal ratio is 1.40625. As mentioned above, the intervals 17:12 and 24:17 (inversions of one another) are very close to an exact half-octave: the decimal ratio of 17:12 is 1.42669. A square with side of 122 has a diagonal of $\sqrt{200}$; 172 is 289, a further confirmation of the closeness of the approximation. As Heath puts it,‡ "Not only did the Pythagoreans discover the irrationality of $\sqrt{2}$, they showed… how to approximate as closely as we please to its numerical value." And, to repeat the point, the calculations involved were directly connected to the representation of musical intervals.

How did the tritone come to be know as *diabolus in musica*? Within the octave, the tritone represents the remotest pitch from that octave's pitch, a sort of "alienation" or exile. Since each whole-tone results from a move of two fifths in a given direction (*c-g-d*, for example), the tritone encompasses six fifths, or in shorthand, $(3/2)^6$,

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* See also "Six Different Tritones" in Harry Partch's *Genesis of a Music*, 2nd ed. (Da Capo, p. 182

† The geometric mean is the square root of the product of the extreme terms, here 1 and 2.

‡ Op. cit. (above, p. 6, note), vol. 1, p. 168.

which is of course the Pythagorean tritone we just saw, 729/512 (with the ratio adjusted by a power of two to remain within the octave). Ernest McClain suggests that 666, "the number of the beast" of the biblical Book of Revelation, may have been a way of writing 3^6, the number of the tritone, "the devil in music."*

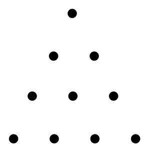
It may be that Plato, too, and the numerological legacy he inherited, influenced our apprehension of the tritone. In his *Republic*, Plato contrasts the life of the king, a just and good ruler, with that of the tyrant (587e): the king's life is 729 times (*enneakaieikosikaiheptakosioplasiakis*) more pleasant (*hedion*) than that of the tyrant, who lives a life the same number of times more painful (*aniarotoron*).

Each musical interval has its own character and history, and could indeed have its own biography: but as a subject the interval of a fourth would certainly rival, or perhaps surpass, any other as most interesting and colorful.

D. LEEDY Corvallis, Oregon • August 2007

* E. McClain: *The Myth of Invariance* (Nicolas Hays, 1976), p. 117. Much of my inspiration and information for the latter part of this essay comes from the work of Ernest McClain, a skilled musician and exemplary scholar of ancient esoteric knowledge of music and number, in the foregoing book and in *The Pythagorean Plato* (Nicolas Hays, 1978), where I learned the significance in Plato of the number 729.

Addendum: The ratios of the Greeks' primary consonances, plus the twelfth (3:1) and double octave (4:1) are represented in the Pythagorean pebble-array called the *tetraclys* ("group of four"), an equilateral triangle comprising the numbers 1, 2, 3 and 4



(also signifying point, line, plane and solid) that sum to the Pythagorean number of perfection, ten. See Greek Musical Writings II, edited and transl. by Andrew Barker (Cambridge U. Pr., 1989), p. 30, 218. The tetraclys was "the oracle of Delphi"; it was "the harmony in which the Sirens sing" and "the whole nature of numbers": Lore and Science in Ancient Pythagoreanism by W. Burkartm transl. by E.L. Minar, Jr. (Harvard U. Pr., 1972), p. 72-3, 186-8, 467-8 (quotes on 18, 467). Cf. also West, op. cit. (above, p. 16, note), p. 235

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